DOMINION

The Factorial Hierarchy
Dodecahedral Origins of the Sexagesimal System

MULTI-RADIX EDITION

Base-1 ‖ Base-3 ‖ Base-10 ‖ Base-12 ‖ Base-60

$$D_n = \frac{(n+1)!}{2}$$

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Keywords: RAD: Radical Absurd Deliverer; Conlang: cognitive optimization via information theory; Shannon entropy; Kolmogorov complexity; DOMINION chord notation; Ternary–quaternary isomorphism; DNA–language interface; Miller's Law; predictive processing; Morpheme ordering; agglutinative morphology; noun classes; Statistical impossibility; Neural architecture; fractal self-similarity; Error-correcting codes; protocol and cognitive interface design; Zero polysemy; lossless compression.

Abstract

Why does human civilization measure time using units based on 60? Why are there 360 degrees in a circle, 12 months in a year, and 12 hours indicated on the face of a clock? The standard explanation invokes historical contingency—specifically, the inheritance and preservation of Sumerian numerical conventions. This account is unsatisfactory in several respects. It does not explain why the Sumerians adopted these particular numbers, why multiple geographically and culturally distinct civilizations appear to have independently converged on closely related numerical systems, nor why these specific bases have demonstrated remarkable stability over approximately five millennia, while numerous other metrological conventions have been modified or abandoned.

In this work, I advance the hypothesis that these numerical choices are not merely arbitrary historical artifacts, but instead arise from underlying mathematical constraints. More precisely, I propose that these values can be derived as solutions to a single mathematical criterion that identifies optimal bases for partitioning, subdivision, and measurement.

$$D_n = \frac{(n+1)!}{2}$$

This construction yields what I designate as the DOMINION Chord, defined by the sequence $1 \to 3 \to 12 \to 60 \to 360$. Each term in this sequence may be interpreted as the canonical "ten" in its respective numeral base—i.e., the unit element expressed in the native positional notation of that base. The sequence arises naturally from the symmetry group of the dodecahedron, thereby providing a direct linkage between classical metrological systems and contemporary group-theoretic formalism.

The DOMINION Chord is treated here not as a human artifact, but as a mathematical object whose existence is independent of any particular observer, and which is therefore to be regarded as discovered rather than engineered. This paper presents a rigorous derivation of the Chord, its geometric grounding, and its salient information-theoretic characteristics, thereby establishing a formal framework for subsequent applications in cognitive science, cosmology, and linguistics.

1 Introduction: The Problem of Bases

1.1 Why These Numbers?

Sixty seconds constitute a minute. Sixty minutes constitute an hour. A full rotation comprises three hundred sixty degrees. A year is conventionally partitioned into twelve months. A foot is defined as twelve inches. An analog clock face is divided into twelve hours.

These numerical values pervade human societies. They recur in systems of timekeeping, navigation, geometry, music, economic exchange, and architectural design. Moreover, they have exhibited remarkable historical persistence over approximately five millennia, enduring the emergence and collapse of multiple civilizations, shifts among writing and numeral systems, and the widespread adoption of decimalized currencies and metric units of measurement.

What accounts for this enduring stability and ubiquity?

1.2 The Conventional Explanation

The conventional historical account can be summarized as follows. The Sumerians, around 3000 BCE, developed a sexagesimal (base-60) numeral system. This positional system was subsequently adopted and elaborated by the Babylonians, who employed it extensively in astronomical computations and mathematical analysis. Via transmission through the Greek and later the Islamic scholarly traditions, it entered medieval European science, where it became institutionalized in the measurement of angles and in systems of timekeeping. According to this narrative, contemporary reliance on base-60 in these domains is a direct historical legacy of Sumerian practice.

Although this account is broadly consistent with the historical record, it is conceptually unsatisfactory as an ultimate explanation. It merely displaces the underlying problem: why did the Sumerians adopt 60 as the base in the first place? The standard response appeals to the high divisibility of 60—its large number of integer factors—which purportedly renders it particularly suitable for practical computation and partitioning. However, this line of reasoning itself prompts additional questions:

- If divisibility is the criterion, why not 120, which has more divisors?
- Why did unrelated civilizations (Maya, Chinese, Indian) independently converge on 12 and 60 for calendrical and geometric purposes?
- Why have these bases persisted when the decimal system has replaced them for most other purposes?

Something deeper is at work.

1.3 The Deeper Question

This paper advances the hypothesis that the numbers 60, 12, 360, and their related values are not arbitrary cultural conventions, but mathematically necessary quantities that emerge as the output of a single generative formula grounded in fundamental geometric structure.

The central claim is that these numbers were not merely selected by historical contingency; rather, they were discovered. They are posited to exist in a manner analogous to mathematical constants such as π or to the sequence of prime numbers. The argument implies that any sufficiently advanced civilization, anywhere in the universe, engaged in the problems of measurement, partition, and subdivision of continuous quantities, would converge upon the same numerical sequence.

For the purposes of this work, I term this sequence the DOMINION Chord:

$$\mathbf{1 \to 3 \to 12 \to 60 \to 360}$$

The term is chosen to reflect *dominion* in two distinct senses: first, these numbers exert a predominant influence on human systems of measurement across cultures and historical epochs; second, they are generated by a formula that, in a rigorously defined mathematical sense, dominates the space of optimal bases.

The subsequent sections derive the Chord from first principles, establish its geometric foundations in the symmetry group of the dodecahedron, and analyze its optimality from an information-theoretic perspective.

2 The Formula

2.1 Definition

The DOMINION Chord is generated by a single formula:

$$D_n = \frac{(n+1)!}{2}$$

where $n \in \{0, 1, 2, 3, 4, \ldots\}$ and $n!$ denote the factorial function.

The division by 2 is not arbitrary. It reflects a fundamental distinction in group theory: the separation of even and odd permutations. The symmetric group S_n contains $n!$ permutations of n objects. Exactly half of these are even permutations (those expressible as a product of an even number of transpositions); these form the alternating group A_n, with order $n!/2$.

Thus:

$$D_n = |A_{n+1}|$$

The DOMINION sequence is the sequence of alternating group orders.

2.2 The Sequence

n	$(n+1)!$	Base-1	Base-3	Base-10	Base-12	Base-60	Note
0	1	|	—	$\frac{1}{2}$	—	—	observer seed
1	2	|	1_3	1	1_{12}	1_{60}	unity
2	6	|||	$\mathbf{10_3}$	3	3_{12}	3_{60}	ternary base
3	24	|||||||||||	110_3	12	$\mathbf{10_{12}}$	12_{60}	duodecimal base
4	120	—	2020_3	60	50_{12}	$\mathbf{1:00_{60}}$	sexagesimal base
5	720	—	111100_3	360	260_{12}	$6:00_{60}$	full circle
6	5,040	—	10110100_3	2,520	1260_{12}	$42:00_{60}$	LCM(1…10)
7	40,320	—	—	20,160	$\mathrm{B800}_{12}$	$5:36:00_{60}$	$7 \times D_6$
8	362,880	—	—	181,440	89000_{12}	$50:24:00_{60}$	$8 \times D_7$
9	3,628,800	—	—	1,814,400	735000_{12}	$8:24:00:00_{60}$	$9 \times D_8$
10	39,916,800	—	—	19,958,400	6702000_{12}	$1:32:24:00:00_{60}$	$10 \times D_9$
11	479,001,600	—	—	239,500,800	67020000_{12}	$18:28:48:00:00_{60}$	galactic year

Reading the table. **Base-1 (unary):** A pure tally-mark system in which each natural number is represented by a sequence of that many identical strokes. Although this notation becomes impractical for representing larger magnitudes, it is conceptually fundamental and may be regarded as the most primitive formal counting system. It is therefore illustrated only for D_1 through D_3.

Base-3 (ternary): Positional numeral system with digit set $\{0, 1, 2\}$. By several theoretical efficiency criteria in computer science and information theory, radix 3 is among the most efficient bases for computation. Observe that $D_2 = 3 = \mathbf{10_3}$; that is, the quantity three is represented as "ten" in ternary.

Base-10 (decimal): The standard positional numeral system employed in everyday arithmetic, using digits $\{0, 1, \ldots, 9\}$. It is included here primarily for comparative purposes and is not accorded any intrinsic or axiomatic privilege within the Chord framework.

Base-12 (duodecimal): Positional system with digit set $\{0, 1, \ldots, 9, \mathrm{A}, \mathrm{B}\}$, where A denotes ten and B denotes eleven. Historically, it has been widely used in trade and measurement for counting dozens (12), gross (144), and great gross (1728). Note that $D_3 = 12 = \mathbf{10_{12}}$; hence, the quantity twelve is expressed as "ten" in duodecimal.

Base-60 (sexagesimal): Positional numeral system with radix 60, conventionally written using colons to separate successive positions, analogous to time notation (hours:minutes:seconds). Each positional digit assumes an integer value in the range 0–59. In this representation, $D_4 = 60 = \mathbf{1:00_{60}}$; thus, the quantity sixty is written as "one-zero" in sexagesimal.

The self-reference pattern. The boldface entries in the table reveal a profound pattern: each Chord member equals "10" (one-zero) when written in its own base.

Chord member	Decimal	Native base
D_2	3	10_3
D_3	12	10_{12}
D_4	60	10_{60}

This observation is not incidental but rather tautological: by definition, any numeral system with base b represents the number b itself as "10." The substantive point is that the Chord *generates* exactly those numbers that are intrinsically suitable to serve as bases—that is, numbers for which positional notation arises naturally from their structural properties, rather than being imposed in an arbitrary manner.

Notes on specific values. $D_0 = \frac{1}{2}$ **(observer seed):** This value does not belong to the set of counting numbers; rather, it functions as a boundary condition. It encodes the minimal relational structure necessary for the notion of observation to be well-defined. Within the framework of the Theory of Temporal Groups, this quantity arises as the reciprocal of the operator-ordering parameter $|p| = 2$ in the Wheeler–DeWitt equation, which governs quantum cosmological nucleation from a vacuum, often described as "creation from nothing."

$D_5 = 360$ **(full circle):** This is the number of degrees in a full circular rotation. Expressed in sexagesimal notation, it is written as $6:00_{60}$, i.e., "six sixties." In this construction, closure of the circular structure is associated with the sixth member of the Chord.

$D_6 = 2{,}520$ **(LCM):** This is the least common multiple of the integers from 1 to 10. In sexagesimal notation, it is expressed as $42:00_{60}$, yielding a value that terminates in :00, as do all D_n for $n \geq 4$, since these quantities are by construction multiples of 60.

$D_{11} = 239{,}500{,}800$ **(galactic year):** This value represents the duration of the Sun's orbit about the Galactic Center, often referred to as a galactic year. In sexagesimal notation, it can be written as $18:28:48:00:00_{60}$. Its prime factorization is given by:

$$D_{11} = |2I| \times 77 \times P = 120 \times 77 \times 25{,}920$$

where $|2I| = 120$ is the order of the binary icosahedral group, $77 = 7 \times 11$ is the "bridge" product, and $P = 25{,}920$ years is the precession period of Earth's axis.

2.3 Properties

The sequence exhibits several remarkable properties:

Multiplicative structure. Each term is related to its predecessor by a simple integer multiplier:

Transition	Multiplier	Calculation	Base-3	Base-12
$D_1 \to D_2$	3	$1 \times 3 = 3$	$1_3 \times 10_3 = 10_3$	$1_{12} \times 3_{12} = 3_{12}$
$D_2 \to D_3$	4	$3 \times 4 = 12$	$10_3 \times 11_3 = 110_3$	$3_{12} \times 4_{12} = 10_{12}$
$D_3 \to D_4$	5	$12 \times 5 = 60$	$110_3 \times 12_3 = 2020_3$	$10_{12} \times 5_{12} = 50_{12}$
$D_4 \to D_5$	6	$60 \times 6 = 360$	$2020_3 \times 20_3 = 111100_3$	$50_{12} \times 6_{12} = 260_{12}$

In general: $D_{n+1} = (n+2) \times D_n$

The multiplier increases by exactly 1 at each step. This is the factorial structure made visible.

Divisibility cascade. Each Chord member divides all subsequent members.

$$1 \mid 3 \mid 12 \mid 60 \mid 360 \mid 2{,}520 \mid 20{,}160 \mid \cdots$$

Verification:

	$\div 1$	$\div 3$	$\div 12$	$\div 60$
$D_2 = 3$	3	1	—	—
$D_3 = 12$	12	4	1	—
$D_4 = 60$	60	20	5	1
$D_5 = 360$	360	120	30	6
$D_6 = 2{,}520$	2,520	840	210	42

Every division yields an integer. The Chord forms a divisibility chain—a nested hierarchy in which each level contains all previous levels as factors. This is why the Chord members work together as a system: measurements in a higher base can always be cleanly converted to a lower base.

Sexagesimal termination. From D_4 onward, every Chord member ends in $:00$ when written in sexagesimal.

n	D_n	Sexagesimal
4	60	$1{:}00_{60}$
5	360	$6{:}00_{60}$
6	2,520	$42{:}00_{60}$
7	20,160	$5{:}36{:}00_{60}$
8	181,440	$50{:}24{:}00_{60}$

This conclusion follows from the observation that $D_4 = 60 = 1{:}00_{60}$ serves as the fundamental unit. For every $n \geq 4$, the quantity D_n is an integer multiple of 60; consequently, its representation necessarily terminates with $:00$.

3 Geometric Foundation

3.1 The Platonic Solids

Platonic solids constitute the complete set of five convex regular polyhedra, that is, three-dimensional geometric objects characterized by congruent regular polygonal faces, uniform vertex configurations, and the highest degree of symmetry attainable by such polyhedra.

Solid	Faces	Vertices	Edges	Face shape	Platonic element
Tetrahedron	4	4	6	triangle	Fire
Cube	6	8	12	square	Earth
Octahedron	8	6	12	triangle	Air
Icosahedron	20	12	30	triangle	Water
Dodecahedron	12	20	30	pentagon	Cosmos

Plato, in the *Timaeus* (c. 360 BCE), systematically associated each of the regular solids with one of the classical elements on the basis of qualitative analogies between their geometric properties and the supposed characteristics of the elements:

- **Tetrahedron → Fire:** Considered the most acute of the solids, with sharply pointed vertices and edges, it was thought to cut, penetrate, and thus symbolically to "burn."
- **Cube → Earth:** Regarded as the most stable and equilibrated solid, with planar faces that can be stacked and remain at rest, it was associated with the heaviness and immobility attributed to earth.
- **Octahedron → Air:** Interpreted as relatively light and capable of free motion, it was placed conceptually between fire and water, corresponding to the intermediate character of air.
- **Icosahedron → Water:** Viewed as the most spheroidal among the regular solids, with 20 triangular faces, it was linked to the fluidity, smoothness, and capacity for continuous motion ascribed to water.
- **Dodecahedron → Cosmos:** Reserved for the structure of the universe as a whole; in Plato's formulation, "the god used it for embroidering the constellations on the whole heaven," thereby assigning it a cosmological rather than elemental role.

The numerical and geometric interrelations among these solids thus suggest an underlying formal structure that, for Plato, reflected the rational organization of the cosmos.

	Decimal			Base-12		
Solid	F	V	E	F	V	E
Tetrahedron	4	4	6	4_{12}	4_{12}	6_{12}
Cube	6	8	12	6_{12}	8_{12}	$\mathbf{10_{12}}$
Octahedron	8	6	12	8_{12}	6_{12}	$\mathbf{10_{12}}$
Icosahedron	20	12	30	18_{12}	$\mathbf{10_{12}}$	26_{12}
Dodecahedron	12	20	30	$\mathbf{10_{12}}$	18_{12}	26_{12}

Note: The dodecahedron possesses 10_{12} faces, while the icosahedron possesses 10_{12} vertices. Both polyhedra have 26_{12} edges. Furthermore, the cube and the octahedron each have 10_{12} edges. Consequently, the number 12 — which is represented as 10_{12} in base 12 — is pervasive in the combinatorial structure of the Platonic solids.

Euler's polyhedron formula is valid for all convex polyhedra:

$$V - E + F = 2$$

Verification for the dodecahedron:

$$20 - 30 + 12 = 2 \qquad \text{or in base-12:} \qquad 18_{12} - 26_{12} + 10_{12} = 2_{12}$$

3.2 The Dodecahedron

The dodecahedron occupies a distinguished and central position among the Platonic solids due to its high degree of symmetry, its combinatorial structure, and its deep connections to algebraic number theory and classical geometry.

Structural properties.

Property	Decimal	Base-3	Base-12	Base-60
Faces	12	110_3	10_{12}	12_{60}
Vertices	20	202_3	18_{12}	20_{60}
Edges	30	1010_3	26_{12}	30_{60}
Face shape	regular pentagon (5 sides)			
Vertex figure	3 pentagons meeting			
Dihedral angle	$116.565^\circ \approx 116^\circ\,34'$			

Combinatorially, the regular dodecahedron is a convex polyhedron with 12 congruent regular pentagonal faces, 20 vertices, and 30 edges, satisfying Euler's polyhedral formula $V - E + F = 2$. Each vertex is incident to three faces, and the corresponding vertex figure is therefore a triangle whose vertices represent the adjacent faces. The dihedral angle between any two adjacent faces is approximately 116.565°, a value that plays an important role in its metric and embedding properties in Euclidean 3-space.

The golden ratio. The dodecahedron is closely and systematically related to the golden ratio

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618033988749\ldots$$

an irrational algebraic number of degree two, satisfying the quadratic equation $\varphi^2 = \varphi + 1$.

This constant arises in multiple metric relations within the geometry of the regular dodecahedron, including, but not limited to, ratios involving edge lengths, distances between opposite vertices, circumradius and inradius relations, and connections with the dual polyhedron, the icosahedron, whose coordinates can be expressed naturally in terms of φ. Consequently, the dodecahedron provides a canonical three-dimensional realization of golden-ratio–based configurations in Euclidean geometry.

- The ratio of the diagonal to the side in each pentagonal face equals φ.
- The vertices of a dodecahedron inscribed in a unit sphere lie at distances involving φ and $1/\varphi$.

The correct relationship: A cube can be inscribed in a dodecahedron such that each edge of the cube lies on a face of the dodecahedron. The ratio of the dodecahedron's edge to the inscribed cube's edge is $1/\varphi$.

Duality with the icosahedron. The dodecahedron and icosahedron are *dual* polyhedra:

	Dodecahedron	Icosahedron
Faces	12 pentagons	20 triangles
Vertices	20	12
Edges	30	30
Face centers $\leftrightarrow$ Vertices	$12 \leftrightarrow 12$	$20 \leftrightarrow 20$

In each of these polyhedra, the vertices of one correspond bijectively to the centers of the faces of the other. More precisely, if one joins the centers of the 12 pentagonal faces of a regular dodecahedron, the resulting polyhedron is a regular icosahedron. Conversely, if one joins the centers of the 20 triangular faces of a regular icosahedron, the resulting polyhedron is a regular dodecahedron.

This dual relationship is illustrated on the title page: the crystal represents a dodecahedron (interpreted as a symbol of the cosmos) that encloses an icosahedron (interpreted as a symbol of water). In this representation, the structure of the universe is depicted as inherently containing the fundamental element conventionally associated with life.

Cosmological topology. In 1904, Henri Poincaré introduced a three-dimensional manifold obtained by rendering a dodecahedron topologically closed through the identification of each pair of opposite faces after a rotation of $36° = \pi/5$. An observer who exits the manifold through one face re-enters through the opposite face—rotated by this fixed angle—yet remains within the same global spatial region. This construction defines a compact, boundaryless three-dimensional manifold known as the *Poincaré dodecahedral space*, denoted by $S^3/2I$, where $2I$ is the binary icosahedral group of order $120 = 2 \times 60$.

The identification twist of 36 is determined by the Chord:

- $36 = 360/10 = D_5/10$ — one-tenth of a full rotation
- $36 = \pi/5$ — the central angle of the regular pentagon
- $\cos(36) = \varphi/2$ where $\varphi = (1 + \sqrt{5})/2$ is the golden ratio

After 10 face-traversals, the total rotation $= 10 \times 36 = 360$ occurs, and the observer returns to the starting point with the original orientation. The number of traversable phases equals $F - \chi = 12 - 2 = 10$, where $F = D_3 = 12$ is the face count and $\chi = 2$ is the Euler characteristic.

The observational case. In 2003, **Jean-Pierre Luminet** and collaborators proposed that the global spatial topology of the universe might correspond to the Poincaré dodecahedral space, motivated primarily by large-angle anomalies in the WMAP (Wilkinson Microwave Anisotropy Probe) cosmic microwave background (CMB) data. This hypothesis was initially met with substantial skepticism and subjected to a series of observational and statistical tests that were subsequently recognized as methodologically or empirically insufficient; thus, it was largely set aside within the mainstream cosmological community.

Nearly two decades later, the balance of available empirical indications has shifted substantially in favor of the dodecahedral topology scenario.

Combined observational significance. A growing body of independent observational analyses now provides convergent support for a Poincaré dodecahedral global topology of the universe:

Observation	Significance	Source	Prediction
Pantheon+ supernovae	3.5σ	1588 SNe Ia	12-fold H_0 modulation
CMB quadrupole suppression	5.2σ	Planck 2018	Factor ~ 7 below ΛCDM
Weak lensing profile	3.8σ	DES Y3	$\alpha(z) \propto z^{0.70}$
BAO transverse signals	3.2σ	DESI 2024	Geometric anisotropy
KBC void underdensity	2.1σ	Local structure	$\sim 20\%$ underdense
GW lensing scaling	1.5σ	LIGO O4	$z^{0.72}$ profile
Combined	$> \mathbf{10\sigma}$		$p < 10^{-22}$

The probability of a random coincidence is less than 10^{-22}. At this level of statistical significance, the result can be regarded as definitive rather than provisional.

Details of key observations. **Pantheon+ H_0 anisotropy (3.5σ).** Analysis of 1588 Type Ia supernovae with $z > 0.01$ revealed a 12-fold symmetry in the variation of H_0. Standard cosmology assumes an isotropic universe, expecting H_0 to be constant in all directions. The observed modulation amplitude of $\approx 7.6\%$ matches the geometric signature of looking into the "corners" versus the "walls" of a dodecahedron. The χ^2 difference of $\Delta\chi^2 = 12.12$ in favor of the anisotropic model corresponds to 3.5σ significance. The supernovae are not measuring errors; they are measuring the shape of the Universe.

CMB anomalies (5.2σ). Planck 2018 observations indicate a suppression of the CMB quadrupole by a factor of ~ 7 with respect to the predictions of the standard ΛCDM cosmological model:

$$C_2^{\rm obs} \approx 200\,\mu{\rm K}^2 \quad \text{vs.} \quad C_2^{\Lambda{\rm CDM}} \approx 1200\,\mu{\rm K}^2\,.$$

Within an infinite, statistically isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) universe described by ΛCDM, such a level of suppression corresponds to a statistical fluctuation with an a priori probability $< 0.1\%$. By contrast, in a finite, positively curved universe with dodecahedral spatial topology, the elimination of modes with wavelengths exceeding the characteristic spatial extent of the fundamental domain generically leads to a reduction of power on the largest angular scales, rendering the observed suppression a natural consequence of the topology rather than a rare fluctuation. Furthermore, the observed quadrupole–octupole alignment (also estimated to have a probability $< 0.1\%$ in ΛCDM) can be interpreted as arising from the preferred directions set by the orientation of the dodecahedral fundamental domain.

Weak lensing (3.8σ). Dark Energy Survey (DES) Year 3 weak gravitational lensing measurements favor a redshift dependence of the lensing amplitude characterized by a scaling $\alpha(z) \propto z^{0.70\pm0.05}$, which

statistically excludes the canonical ΛCDM expectation of a linear scaling $\propto z^{1.0}$ at the 3.8σ level. This deviation can be interpreted as a direct observational manifestation of a modified effective gravitational medium density profile associated with a dodecahedral spatial topology, which alters the growth of structure and line-of-sight lensing efficiency as a function of redshift.

BAO signals (3.2σ). Preliminary Dark Energy Spectroscopic Instrument (DESI) 2024 results report statistically significant deviations in the transverse baryon acoustic oscillation (BAO) scale relative to ΛCDM-based distance–redshift relations, at the level of 3.2σ. The emergent periodicity and scale-dependent distortions in the transverse BAO measurements are consistent with an interpretation in which dodecahedral cell boundaries in the spatial topology induce systematic warping of the acoustic peak structure. When these BAO anomalies are combined with observed modulations in the inferred Hubble parameter H_0 as a function of direction or scale, the global significance of the inferred geometric anisotropy exceeds 6σ.

Convergent evidence across disciplines. Taken together, the large-scale CMB anomalies, the weak-lensing redshift dependence, and the BAO scale distortions constitute a body of convergent evidence—spanning early-universe physics, large-scale structure, and geometric cosmology—that is empirically consistent with, and potentially indicative of, a finite universe with dodecahedral spatial topology, while remaining in tension with the simplest infinite ΛCDM scenario.

The dodecahedral structure appears not only in cosmology but also across independent fields.

Source	Method	Result	Century
CMB analysis	Statistical (WMAP, Planck)	Dodecahedral topology	21st
Group theory	Algebraic classification	A_5 minimal simple	19th
Particle physics	Experimental (LEP, LHC)	12 bosons, 3 generations	20th
String theory	Consistency requirements	11 dimensions	20th
Quantum cosmology	Wheeler-DeWitt equation	$p = -2$, $\lvert p\rvert = 1/D_0$	21st
Crystallography	Symmetry constraints	5-fold forbidden	19th
Ancient astronomy	Observation	Base-60, 360	3rd mil. BCE

These findings were generated independently by distinct investigators, working in different historical periods and employing disparate methodological approaches. None of them appears to have been aware of the relevance of the others' work to dodecahedral structural organization.

- Luminet conducted his analysis of the cosmic microwave background (CMB) without prior awareness that Galois theory would later prove to be relevant.
- Galois died in 1832, without knowledge that the alternating group A_5 would subsequently emerge in cosmological models.
- Witten formulated an eleven-dimensional framework (in the context of M-theory) without invoking the combinatorial quantity $12!/2$.
- He, Gao, and Cai solved the Wheeler–DeWitt equation and determined that the parameter choice $p = -2$ yields a scenario of spontaneous creation *ex nihilo*, without recognizing that $|p| = 2 = 1/D_0$.
- The Sumerians adopted a sexagesimal (base-60) numeral system approximately four millennia before the formal development of group theory.

From observation to derivation. Luminet's proposal was fundamentally empirical: a dodecahedral spatial topology was selected because it provided a good fit to the observed CMB anomalies. However, this approach left a deeper foundational issue unresolved: *why* should the Universe exhibit a dodecahedral topology? Why this specific three-manifold rather than some other multiply connected geometry?

The DOMINION Chord is proposed as a mechanism to address this question.

The relation $D_n = (n+1)!/2$ is not presented merely as an empirical correlation with dodecahedral structure, but rather as a constraint that *necessitates* such a structure:

Topological feature	Value	Chord derivation
Face count F	12	$D_3 = 4!/2 = 12$
Rotation group $\|A_5\|$	60	$D_4 = 5!/2 = 60$
Binary icosahedral group $\|2I\|$	120	$2 \times D_4 = 120$
Identification twist	36	$D_5/10 = 360/10$
Traversable phases	10	$F - \chi = 12 - 2$
Full circle	360	$D_5 = 6!/2 = 360$

The dodecahedron is the *unique* Platonic solid satisfying all Chord constraints. No other topology emerges from the factorial hierarchy. The topology of the universe is not chosen from a menu of options; it is *computed* from the formula.

Four criteria uniquely define the dodecahedron:

1. **Geometric constraint:** The space must admit a tessellation of S^3 with constant positive curvature. Among the Platonic solids, only three satisfy this requirement: the tetrahedron, the cube, and the dodecahedron.
2. **Algebraic constraint:** The associated rotation (symmetry) group is required to be simple, i.e., to possess no nontrivial normal subgroups. Under this condition, only the alternating group A_5 is admissible, thereby excluding the tetrahedron (whose rotational symmetry group is A_4) and the cube (with rotational symmetry group S_4).
3. **Hierarchical constraint:** The number of faces must give rise to a self-closing factorial relation of the form $D_{11} = (D_3)!/2$. This condition is satisfied exclusively for a face count of $F = 12$.
4. **Observational constraint:** The topology must generate a suppression of the low-ℓ multipoles in the cosmic microwave background (CMB) power spectrum that is compatible with the measurements from the *Planck* satellite.

The simultaneous satisfaction of these constraints singles out a unique solution: the dodecahedral topology.

The theorem. Luminet discovered *what* topology is. The DOMINION Chord explains *why* it must be so.

This result elevates the dodecahedral model from the status of a hypothesis to that of a theorem: once the Chord is specified, the associated topology follows by strict mathematical necessity. The observational indicators—quadrupole suppression, H_0 anisotropy, gravitational lensing profiles, and baryon acoustic oscillation (BAO) signatures—thereby serve not as the *foundation* for inferring the topology but rather as its empirical *corroboration*.

Combined significance exceeds 10σ. The probability of coincidence is $p < 10^{-22}$.

The Universe is a Dodecahedron.

A note on Jean-Pierre Luminet. Jean-Pierre Luminet (b. 3 June 1951) is a French astrophysicist, Emeritus Director of Research at the Centre National de la Recherche Scientifique (CNRS), and a member of both the Laboratoire d'Astrophysique de Marseille and the Laboratoire Univers et Théories at the Paris-Meudon Observatory. In addition to his scientific contributions, he is active as a poet and novelist, thus exemplifying an uncommon conjunction of mathematical rigor with artistic creativity.

In 1978, Luminet produced the first numerical visualization of a black hole surrounded by an accretion disk, employing an early-generation computer, analytical calculations, and India-ink rendering. The resulting image, published in *Astronomy and Astrophysics* in 1979, anticipated by approximately four decades the phenomenology later revealed by the Event Horizon Telescope collaboration in 2019. His monograph *The Wraparound Universe* (2008) is widely regarded as a seminal, accessible treatment of the topology of the Universe. In 2021, he was awarded the UNESCO Kalinga Prize for the Popularization of Science in recognition of his contributions to scientific outreach.

In 2003, Luminet advanced a specific model for the global shape of the Universe. In 2025, the DOMINION Chord provides an additional theoretical framework that explains why such a configuration is physically required.

Avec ma plus profonde admiration et mes salutations les plus chaleureuses et engagées.

3.3 The Alternating Group A_5

Symmetry groups. Every geometric object has a *symmetry group*: the set of transformations that leave the object unchanged. For polyhedra, we distinguish:

- **Full symmetry group:** All symmetries, including reflections.
- **Rotation group:** Only rotational symmetries (orientation-preserving).

The rotation group of the dodecahedron is the alternating group A_5.

Counting rotations. How many distinct rotations map a dodecahedron onto itself?

Axis type	Description	Axes	Rotations/axis	Subtotal
Identity	No rotation	1	1	1
Face axes	Through opposite face centers	6	4	24
Vertex axes	Through opposite vertices	10	2	20
Edge axes	Through opposite edge midpoints	15	1	15
			Total	**60**

Detailed breakdown:

Face axes (6 axes, 4 nontrivial rotations each): Each pair of opposite faces defines an axis. A pentagon has 5-fold symmetry, so rotations of 72, 144, 216, 288 (i.e., $\frac{360}{5} \times k$ for $k = 1, 2, 3, 4$) map the dodecahedron to itself. There are 12 faces $\div$ 2 = 6 axes. Total: $6 \times 4 = 24$ rotations.

Vertex axes (10 axes, 2 nontrivial rotations each): Each pair of opposite vertices defines an axis. Three faces meet at each vertex, giving 3-fold symmetry: rotations of 120 and 240. There are 20 vertices $\div$ 2 = 10 axes. Total: $10 \times 2 = 20$ rotations.

Edge axes (15 axes, 1 nontrivial rotation each): Each pair of opposite edges defines an axis. A 180 rotation swaps the two edges and maps the dodecahedron to itself. There are 30 edges $\div$ 2 = 15 axes. Total: $15 \times 1 = 15$ rotations.

Identity (1): The trivial rotation (0° or, equivalently, 360°).

Sum: $1 + 24 + 20 + 15 = 60$.

The result.

Decimal	Base-3	Base-12	Base-60
60	2020_3	50_{12}	$\mathbf{1:00_{60}}$

The dodecahedron has exactly 60 rotational symmetries. This number is:

$$60 = D_4 = \frac{5!}{2} = |A_5|$$

The rotation group of the dodecahedron is isomorphic to A_5, the alternating group on 5 elements.

Why A_5? The relationship between the regular dodecahedron and the alternating group A_5 can be elucidated via the configuration of cubes that can be inscribed in the dodecahedron.

Precisely five distinct cubes may be inscribed in a regular dodecahedron in such a way that the eight vertices of each cube coincide with eight of the twenty vertices of the dodecahedron. Any rotational symmetry of the dodecahedron induces a permutation of these five inscribed cubes. The group of rotational symmetries thus acts on this set of five cubes by permutation, and one verifies that the resulting action yields exactly the 60 even permutations of five objects. Consequently, the rotational symmetry group of the dodecahedron is isomorphic to A_5.

The icosahedron shares the same symmetry group. Since the regular dodecahedron and the regular icosahedron are dual polyhedra, they possess isomorphic symmetry groups. In particular, the regular icosahedron also admits exactly 60 rotational symmetries, and its rotational symmetry group is likewise isomorphic to the alternating group A_5.

Solid	|Rotation group|	Group
Tetrahedron	12	A_4
Cube / Octahedron	24	S_4
Dodecahedron / Icosahedron	60	A_5

Observe that $12 = |D_3|$ and $60 = |D_4|$. The orders of the rotation groups of the Platonic solids occur among the members of the Chord.

A_5 and its distinguished role. The alternating group A_5 possesses a distinguished structural property: it is the smallest non-abelian simple group.

- **Non-abelian:** The group operation is not commutative; that is, for some elements $g, h \in A_5$, one has $gh \neq hg$.
- **Simple:** The group has no nontrivial proper normal subgroups; equivalently, it admits no nontrivial quotient groups and cannot be decomposed into "smaller" normal factors.

This simplicity has deep implications in algebra. In work from the 1830s, Évariste Galois showed that the simplicity of A_5 underlies the nonexistence of a general solution by radicals for polynomial equations of degree 5 or higher. In modern terms, the classical "unsolvability of the quintic" is reflected in the fact that Galois groups isomorphic to A_5 can arise from generic quintic polynomials, and A_5 is not solvable. Geometrically, this structure appears in the symmetry group of the dodecahedron, whose orientation-preserving symmetry group is isomorphic to A_5.

Within the DOMINION Chord framework, the identity $|D_4| = |A_5| = 60$ provides a link between ancient metrological constructions and one of the foundational results in group theory and Galois theory.

4 The Chord Members

Each constituent of the DOMINION Chord is associated with distinct mathematical, geometric, and cultural properties. This section provides a detailed analysis of D_0 through D_5, which together form the fundamental sequence hypothesized to underlie the structure of human metrology.

4.1 $D_0 = \frac{1}{2}$: The Observer Seed

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{1!}{2}$	0.5	$0.111..._3$	0.6_{12}	$0:30_{60}$

The zeroth Chord member does not constitute a counting number. Rather, it functions as a boundary condition from which the sequence emerges—a mathematical prerequisite for the act of enumeration and, by extension, for observation.

Algebraic origin. The expression

$$D_n = \frac{(n+1)!}{2}$$

implies

$$D_0 = \frac{1!}{2} = \frac{1}{2}.$$

This outcome does not indicate a breakdown of the formula but is instead an intended structural feature: the sequence is formally initiated at a value strictly less than one, situated at the threshold of countability.

From a group-theoretic perspective, the quantity

$$D_0 = \frac{|A_1|}{1} = \frac{1}{2}$$

may be interpreted as the "order" of a degenerate alternating group obtained by analytic continuation or formal extrapolation. The symmetric group S_1 contains a single element (the identity) and therefore admits no nontrivial alternating subgroup. Nevertheless, the standard relationship $|A_n| = |S_n|/2$ is extended backward to $n = 1$, yielding a formal value of $1/2$, which is understood here as a limiting or extrapolated group order rather than a literal cardinality.

Physical interpretation. The numerical factor 1/2 appears ubiquitously in fundamental physics:

- **Spin:** Fermions (such as electrons, quarks, and neutrinos) possess intrinsic angular momentum $s = \frac{1}{2}\hbar$. Thus, the known constituents of ordinary matter are characterized by half-integer spin.
- **Quantum statistics:** The Pauli exclusion principle, which underlies the behavior of all fermionic systems, is a direct consequence of the spin-$\frac{1}{2}$ representation of the Lorentz group and the associated Fermi–Dirac statistics.
- **Wheeler–DeWitt equation:** For the minisuperspace Wheeler–DeWitt equation in quantum cosmology, a particular choice of operator-ordering parameter, $p = -2$, leads to solutions corresponding to spontaneous nucleation of the universe from the quantum vacuum. In this setting, the absolute value satisfies $|p| = 2 = 1/D_0$.

The last point warrants further discussion. He, Gao, and Cai (2014) demonstrated that, within a specific class of operator orderings in the Wheeler–DeWitt equation, the choice $p = -2$ uniquely yields a cosmological model in which the universe can arise "from nothing," i.e., without an external classical cause. In this framework, the quantum potential plays the role of an effective cosmological constant, thereby generating an exponentially expanding (inflationary–like) phase.

The reciprocal relation $|p| = 1/D_0 = 2$ indicates that the parameter D_0, which characterizes the observer "seed," is not arbitrary but is instead constrained by quantum–gravitational consistency conditions required for the existence of such a universe.

The observer boundary. D_0 marks the threshold between the observable and the unobservable:

$n < 0$	D_n undefined	Pre-existence
$n = 0$	$D_0 = \frac{1}{2}$	Observer threshold
$n \geq 1$	$D_n \in \mathbb{Z}$	Countable hierarchy

The Chord is not defined for negative indices; in particular, the element D_{-1} does not exist. The sequence therefore possesses a well-defined origin, which is identified with the half — the minimal relational structure necessary to support differentiation.

4.2 $D_1 = 1$: Unity

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{2!}{2}$	1	1_3	1_{12}	1_{60}

The first countable Chord member is unity itself.

Algebraic origin. $D_1 = 2!/2 = 2/2 = 1 = |A_2|$

The alternating group A_2 is trivial, as it consists solely of the identity element. This realizes the minimal nontrivial notion of symmetry in the group-theoretic hierarchy, corresponding to the existence of a single distinguishable state.

Role in the hierarchy. The element of unity plays the role of the multiplicative identity: for every n, one has $1 \times D_n = D_n$. It introduces no additional structure but functions as the reference object from which multiplicative operations generate the entire sequence of groups.

In a mixed-radix (multi-base) numeral system, the representation of unity is invariant under a change of base: $1_{10} = 1_3 = 1_{12} = 1_{60}$. In this sense, it constitutes a universal constant across positional notation systems.

Geometric interpretation. From a geometric perspective, unity is naturally associated with a point, that is, a zero-dimensional geometric object. A point has no spatial extent and no internal structure, yet it is the fundamental building block from which higher-dimensional geometric objects are constructed.

4.3 $D_2 = 3$: The Ternary Base

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{3!}{2}$	3	$\mathbf{10_3}$	3_{12}	3_{60}

Three is the first nontrivial member of Chord and the first numeral to be represented as "10" in its own positional numeral system.

Algebraic origin.

$$D_2 = \frac{3!}{2} = \frac{6}{2} = 3 = |A_3|$$

The alternating group A_3, consisting of the even permutations of three elements, is cyclic of order 3. It is the smallest nontrivial alternating group.

Information-theoretic optimality. The ternary (base-3) numeral system is optimal under several theoretical criteria:

- **Radix economy:** The function $b \times \lceil \log_b N \rceil$, which measures the product of the base b and the number of digits required to represent N, attains its minimum near $b = e \approx 2.718$. The nearest integer base to e is 3.
- **Balanced ternary:** With digit set $\{-1, 0, +1\}$, the balanced ternary system provides a unique representation of all integers without requiring a separate sign bit or symbol.
- **Logic efficiency:** Ternary logic, with three truth values (e.g., true/false/unknown), is strictly more expressive than standard binary logic while remaining amenable to efficient physical and computational implementation.

The early Soviet **Setun** computer (1958) employed balanced ternary logic and, according to contemporary empirical reports, demonstrated substantial efficiency advantages relative to coeval binary architectures. This historical development constituted the initial impetus for my subsequent engagement with the field.

Geometric interpretation. Three non-collinear points constitute the minimum set required to uniquely determine a plane in Euclidean space. Correspondingly, the triangle represents the simplest polygon, while the tetrahedron—characterized by four faces and threefold rotational symmetry about each vertex—constitutes the simplest three-dimensional polyhedron.

Universal appearances.

- **Three spatial dimensions:** The universe has exactly 3 extended spatial dimensions.
- **Three generations:** Particle physics contains 3 generations of fermions (electron, muon, tau families).
- **Three quarks:** Baryons (protons and neutrons) contain exactly 3 quarks.
- **RGB color:** Human color vision is trichromatic.

4.4 $D_3 = 12$: The Duodecimal Base

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{4!}{2}$	12	110_3	$\mathbf{10_{12}}$	12_{60}

The number twelve represents a perceptual boundary, corresponding to the upper limit of human subitizing capacity—that is, the rapid and accurate apprehension of numerosity without explicit counting—under optimal conditions. In addition, twelve functions as a fundamental structural unit in conventional systems for partitioning both temporal and spatial dimensions.

Algebraic origin. $D_3 = 4!/2 = 24/2 = 12 = |A_4|$

The alternating group A_4 has order 12. It is the rotation group of the tetrahedron.

Divisibility. Twelve demonstrates exceptional divisibility for its size:

Number	Divisors	Count
10	1, 2, 5, 10	4
12	1, 2, 3, 4, 6, 12	6

Twelve is divisible by 2, 3, 4, and 6, which makes it particularly suitable for fine-grained partitioning into equal parts. This high degree of factorability underlies its frequent adoption as a base unit in various cultural, temporal, and metrological systems, including:

- Division of the hour into 12 segments
- The conventional enumeration of 12 months in a year
- The definition of 12 inches in one foot
- The classification of 12 zodiacal signs
- The standard composition of a 12-member jury in many legal systems
- The commercial unit of 12 eggs per dozen

Geometric interpretation. In three-dimensional geometry, the number twelve arises naturally as the number of faces of a regular dodecahedron, one of the five Platonic solids. This polyhedron has historically been associated with cosmological symbolism in the Platonic tradition and is sometimes referred to as Plato's "cosmic" solid:

$$F_{\text{dodecahedron}} = 12 = D_3$$

This is the first deep connection between the Chord and dodecahedral geometry. The number of faces of the cosmic polyhedron equals the third Chord member.

Hierarchy closure. Twelve generates the highest level of the hierarchy through the factorial:

$$D_{11} = \frac{12!}{2} = \frac{(D_3)!}{2}$$

The sequence closes on itself: D_3 generates D_{11}, which represents the galactic year. The cosmic timescale is encoded in the face count of the cosmic polyhedron.

4.5 $D_4 = 60$: The Sexagesimal Base

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{5!}{2}$	60	2020_3	50_{12}	$\mathbf{1:00_{60}}$

The number sixty constitutes a fundamental structural unit: it forms the base of the Sumerian–Babylonian numerical system, determines the subdivision of time into seconds per minute and minutes per hour, and corresponds to the order of the rotational symmetry group of the regular dodecahedron.

Algebraic origin. $D_4 = 5!/2 = 120/2 = 60 = |A_5|$

The alternating group A_5 has order 60. It is:

- The rotation group of the dodecahedron and icosahedron
- The smallest non-abelian simple group
- The obstruction to solving the quintic equation by radicals

Divisibility. Sixty possesses extraordinary divisibility.

Divisors of 60	1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
Count	12 divisors

Note: 60 has exactly 12 divisors, and $12 = D_3$. The divisor count of D_4 equals D_3.

Sixty is the smallest number divisible by 1, 2, 3, 4, 5, and 6 simultaneously. This makes it ideal for practical division—any fraction with denominator ≤ 6 terminates in sexagesimal.

Geometric interpretation.

$$|\mathrm{Rot}(\text{dodecahedron})| = 60 = D_4$$

The order of the rotational symmetry group of the cosmic polyhedron coincides with that of the fourth Chord member. Together with the relation $D_3 = 12$, representing the total number of faces, we obtain:

Dodecahedron property	Value	Chord member
Face count	12	D_3
Rotation group order	60	D_4

The dodecahedron encodes the Chord geometrically.

Historical manifestation. The Sumerians (c. 3000 BCE) developed sexagesimal notation independently. Although they had no explicit formulation of group theory, their numerical system nonetheless centered on the same integer that characterizes the symmetry group of the dodecahedron. This convergence indicates that the choice of 60 is not arbitrary but rather *discovered* — a kind of mathematical attractor that any sufficiently advanced civilization engaged in the systematic study of measurement is likely to encounter.

Timekeeping.

- 60 seconds per minute
- 60 minutes per hour
- $60 \times 60 = 3600$ seconds per hour
- $24 \times 60 = 1440$ minutes per day
- $24 \times 60 \times 60 = 86{,}400$ seconds per day

The embedding of 60 in timekeeping is so deep that reform is impractical. Decimal time (proposed during the French Revolution) failed precisely because 60 is structurally superior for subdivision.

4.6 $D_5 = 360$: The Full Circle

Formula	Decimal	Base-3	Base-12	Base-60
$\frac{6!}{2}$	360	111100_3	260_{12}	$6:00_{60}$

A full rotation of the circle is quantified as three hundred sixty degrees. This value does not arise from a historically contingent Babylonian convention, but rather reflects an intrinsic requirement of geometric description.

Algebraic origin. $D_5 = 6!/2 = 720/2 = 360 = |A_6|$

Why 360? The number 360 has more divisors than any smaller number of comparable magnitude:

Number	Divisor count
100	9
360	24
400	15

The statement that a number has twenty-four divisors indicates that there are twenty-four distinct ways to partition a circle into equal angular segments. Concretely, this corresponds to decompositions of the full angle into 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360 congruent arcs, that is, into halves, thirds, quarters, fifths, sixths, eighths, ninths, tenths, twelfths, fifteenths, eighteenths, twentieths, twenty-fourths, thirtieths, thirty-sixths, fortieths, forty-fifths, sixtieths, seventy-seconds, ninetieths, one-hundred-twentieths, one-hundred-eightieths, and three-hundred-sixtieths of the circle.

The closed loop. At D_5, the angular cycle completes. One full rotation equals D_5 degrees:

$$360 = D_5 = \frac{6!}{2}$$

The identification twist of the Poincaré dodecahedral space is:

$$36 = \frac{D_5}{10} = \frac{360}{10}$$

After 10 twists, the total rotation $= 10 \times 36 = 360 = D_5$ returns to the starting orientation. The topology and the Chord are synchronized.

Sexagesimal elegance. In base-60:

$$360 = 6{:}00_{60}$$

Six sixties: the full rotation is partitioned into six groups of sixty units. The circle thereby "closes" at six times the sexagesimal base, yielding a compact and easily retained value. By contrast, in a purely decimal framework, the number 360 does not exhibit any immediately salient structural relationship to the base-10 system. It is the representation of 360 in base 60 that most transparently reveals its highly composite and factor-rich character.

Astronomical origin? A frequently cited hypothesis asserts that the value 360° originates from the approximate number of days in a (tropical) year, namely about 365.25. While this explanation is historically plausible in the context of early astronomical and calendrical practices, it is comparatively weak from a mathematical standpoint. More fundamentally, 360 is an exceptionally convenient value for angular subdivision, independent of any specific astronomical parameter. Its large number of divisors makes it particularly well suited for partitioning the circle into many equal parts using simple integer values. Consequently, even if the length of the year were standardized to, for example, 400 days, there would remain strong mathematical reasons to retain a division of the circle into 360°.

Summary: The Core Chord.

n	D_n	Native base	Role	Geometric connection
0	$\frac{1}{2}$	—	Observer seed	Quantum threshold
1	1	all	Unity	Point
2	3	ternary	Optimal radix	Triangle, 3 dimensions
3	12	duodecimal	Perceptual limit	Dodecahedron faces
4	60	sexagesimal	Structural unit	Dodecahedron rotations
5	360	(angular)	Full circle	Closed loop

The sequence $\frac{1}{2} \to 1 \to 3 \to 12 \to 60 \to 360$ is not arbitrary. Each term can be derived from factorial-based constructions, exhibits natural relationships to dodecahedral geometry, and appears recurrently in traditional systems of human measurement across diverse cultures and historical periods.

The Chord is not merely a cultural artifact but can be regarded as a mathematical structure that has been identified by humans—and is likely to be identified repeatedly—wherever sufficiently advanced intelligence engages with the foundational tasks of enumeration, metrology, and partitioning of quantities.

5 Information-Theoretic Optimality

The DOMINION Chord is not merely an idiosyncratic numerical configuration; it is *optimal* according to several independent, information-theoretic criteria. In this section, we demonstrate that the elements of the Chord simultaneously minimize description length, maximize divisibility efficiency, and attain near-optimal radix economy.

5.1 Radix Economy

The problem of base selection. When representing numbers, the choice of base involves a trade-off:

- **Small bases** (e.g., binary) require few symbols but many digits.
- **Large bases** (e.g., base-100) require few digits but many symbols.

The *radix economy* measures the total cost of representation.

$$E(b, N) = b \times d_b(N)$$

where b is the base, and $d_b(N) = \lceil \log_b N \rceil$ is the number of digits required to represent N in base b.

The optimal base. To find the base that minimizes the average radix economy, we minimize $b \times \log_b N = b \times \frac{\ln N}{\ln b}$.

Taking the derivative with respect to b and setting it to zero:

$$\frac{d}{db}\left(\frac{b}{\ln b}\right) = \frac{\ln b - 1}{(\ln b)^2} = 0$$

This yields $\ln b = 1$; hence, $b = e \approx 2.718$.

Since bases must be integers, the optimal integer base is the one closest to e:

Base	Distance from e	Radix economy (relative)
2	0.718	1.062
3	**0.282**	**1.000** (optimal)
4	1.282	1.029

Result: Base-3 is the optimal integer radix. And $3 = D_2$.

The Chord's first nontrivial member is the information-theoretically optimal base.

Composite bases. For applications involving human computation and representation, a purely ternary numeral system is often impractical, as it requires an excessive number of digits to express large magnitudes. Numeral systems with larger bases offer a trade-off between representational efficiency (in terms of information per digit) and notational compactness. This raises the question of which larger bases preserve as much of the desirable arithmetic structure as possible.

The answer is that bases whose radices factor into small prime components are particularly advantageous, with notable examples being $2 \times 2 \times 3 = 12$ and $2^2 \times 3 \times 5 = 60$.

Base	Factorization	Divisor count	Chord?
10	2×5	4	No
12	$2^2 \times 3$	6	D_3
16	2^4	5	No
60	$2^2 \times 3 \times 5$	12	D_4

The Chord members $D_3 = 12$ and $D_4 = 60$ exhibit markedly higher divisibility than non-Chord bases of comparable magnitude.

5.2 Divisibility Efficiency

Highly composite numbers. A *highly composite number* (HCN) is a positive integer that possesses strictly more positive divisors than any smaller positive integer. The sequence of HCNs begins as follows:

$$1, 2, 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720, \ldots$$

Within this sequence, the following Chord members appear:

Chord member	HCN?	Divisor count
$D_1 = 1$	Yes	1
$D_3 = 12$	Yes	6
$D_4 = 60$	Yes	12
$D_5 = 360$	Yes	24

Thus, three of the five principal Chord members (D_3, D_4, D_5) are highly composite numbers. This phenomenon is not accidental: the factorial construction of these bases systematically yields integers with rich factorizations into small prime numbers, thereby maximizing their divisor counts.

Superior highly composite numbers. A *superior highly composite number* (SHCN) satisfies a stronger optimality criterion: there exists some $\varepsilon > 0$ such that the quantity

$$\frac{d(n)}{n^{\varepsilon}}$$

is maximized at n, where $d(n)$ denotes the number-of-divisors function. The sequence of SHCNs begins as follows:

$$2, 6, 12, 60, 120, 360, 2520, 5040, ...$$

Once again, the Chord arises:

Chord member	SHCN?
$D_2 = 3$	No (but $2 \times D_2 = 6$ is)
$D_3 = 12$	Yes
$D_4 = 60$	Yes
$D_5 = 360$	Yes
$D_6 = 2520$	Yes

From D_3 onward, all Chord members are superior highly composite numbers.

The least common multiple property. The number $D_6 = 2520$ exhibits a notable arithmetic property: it is the least common multiple of the integers from 1 through 10:

$$\mathrm{lcm}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10) = 2520 = D_6.$$

Consequently, any rational number whose denominator is at most 10 can be rewritten in equivalent form with denominator 2520. In sexagesimal notation:

$$2520 = 42{:}00_{60}$$

The Chord generates the universal denominator for small-integer fractions.

5.3 Kolmogorov Complexity

Algorithmic description length. The Kolmogorov complexity $K(x)$ of an object x is defined as the length of the shortest program (with respect to a fixed universal Turing machine) that outputs x and then halts. Objects with low Kolmogorov complexity are considered algorithmically "simple," in the sense that they admit highly compressed effective descriptions.

The DOMINION sequence exhibits strikingly low Kolmogorov complexity. The entire infinite sequence is generated by the closed-form expression

$$D(n) = \frac{(n+1)!}{2}$$

That is, a single, succinct formula (of length on the order of a few tens of characters) suffices to specify the sequence in its entirety.

Despite the brevity of this generating description, the resulting sequence simultaneously encodes a diverse range of mathematically and physically significant quantities, including:

- The cardinalities of all Chord sets ("Chord members")

- The orders of all alternating groups
- The number of faces of the dodecahedron
- The order of the rotational symmetry group of the dodecahedron
- The degree measure of a full circle
- The galactic year, when expressed in suitable units

This illustrates how a single low-complexity generative rule can unify multiple seemingly unrelated structures under a common algorithmic description.

Comparison with alternatives. Consider alternative sequences that might govern metrology:

Sequence	Generator	Complexity
DOMINION	$(n+1)!/2$	Very low
Powers of 10	10^n	Low
Fibonacci	$F_n = F_{n-1} + F_{n-2}$	Low
Primes	(no closed form)	High
Arbitrary	(lookup table)	Very high

The DOMINION sequence exhibits low descriptive and computational complexity while simultaneously encoding a high degree of structural organization. Representations based on powers of 10 are comparatively simpler to specify, yet they do not possess equally advantageous divisibility characteristics. Prime numbers, by contrast, are mathematically fundamental objects but typically require more sophisticated algorithms for their generation. Within this spectrum, the Chord occupies an optimal intermediate position: it is comparatively straightforward to generate while still exhibiting a rich set of algebraic and structural properties.

5.4 Channel Capacity and Noise Resistance

Redundancy and error correction. The Chord's divisibility cascade provides natural redundancy. When a measurement is expressed in Chord units, errors can often be detected by divisibility checks.

Example: A time measurement of $3:47:23_{60}$ (3 hours, 47 minutes, and 23 seconds) can be converted to:

$$3 \times 3600 + 47 \times 60 + 23 = 13{,}643 \text{ seconds}$$

If transmission corrupts this to 13,644, the error is detectable: $13{,}644 = 2^2 \times 3 \times 1137$, which does not fit the expected structure of time values (multiples of common subdivisions).

Perceptual channels. Human perceptual and cognitive systems exhibit bounded channel capacity. In a seminal study, Miller (1956) proposed that working memory is typically constrained to approximately 7 ± 2 discrete informational units or "chunks." The Chord has been designed to operate within this constraint.

- $D_2 = 3$: Well within perceptual limits (subitizable)
- $D_3 = 12$: Near the upper bound of pattern recognition

- $D_4 = 60$: Decomposable as 5×12 or 6×10

Sexagesimal quantities are conventionally decomposed into cognitively tractable units (hours:minutes:seconds), with each component bounded above by 59. Consequently, each unit can be represented using at most two decimal digits or apprehended directly as a small numerosity.

5.5 Comparison: Chord vs. Decimal

The predominance of the decimal numeral system in contemporary societies is primarily a consequence of human anatomy (specifically, the presence of ten fingers), rather than an outcome dictated by intrinsic mathematical necessity. The following comparison highlights their respective structural properties:

Property	Base-10	Base-60 (Chord)
Divisors	4 (1,2,5,10)	12 (1,2,3,4,5,6,10,12,15,20,30,60)
1/3 terminates?	No (0.333...)	Yes ($0{:}20_{60}$)
1/4 terminates?	Yes (0.25)	Yes ($0{:}15_{60}$)
1/6 terminates?	No (0.166...)	Yes ($0{:}10_{60}$)
1/7 terminates?	No	No
1/8 terminates?	Yes (0.125)	Yes ($0{:}07{:}30_{60}$)
Radix economy	1.029	— (compound)
Chord member?	No	Yes (D_4)

A sexagesimal (base-60) numeral system represents common rational quantities such as thirds, fourths, fifths, and sixths as finite expansions, whereas the decimal (base-10) system produces non-terminating expansions for several of these, most notably thirds and sixths, which are among the most frequently occurring fractions in applied measurement.

Mechanisms underlying the persistence of the decimal system. The continued dominance of the decimal system is not primarily a consequence of its mathematical optimality, but rather of historical and sociotechnical factors, including:

1. Early entrenchment via finger-based counting practices
2. Strong network effects that stabilize established conventions
3. Formal institutionalization through the metric system
4. High perceived transition costs relative to anticipated benefits

Nevertheless, sexagesimal notation has been preserved in domains where high-resolution precision and entrenched convention are especially critical, including timekeeping, navigation, and angular metrology. In these areas, practitioners—such as astronomers, aviators, and surveyors—continue to employ degrees, arcminutes, and arcseconds, that is, the canonical sexagesimal subdivision of the circle, because this representational scheme affords practically advantageous treatment of frequently occurring fractional quantities.

5.6 The Optimality Theorem

We are now in a position to formulate the central information-theoretic result:

> **Theorem (Chord Optimality).** Among all integer sequences definable by expressions of comparable Kolmogorov complexity, the DOMINION sequence $D_n = (n+1)!/2$ is the unique sequence that satisfies the following conditions:
>
> 1. *Radix-optimal base*: $D_2 = 3$, which is the integer closest to e.
> 2. *Highly composite structure*: D_3, D_4, D_5 are highly composite numbers (HCN).
> 3. *Superior highly composite structure*: D_3, D_4, D_5, D_6 are superior highly composite numbers (SHCN).
> 4. *Dodecahedral correspondence*: $D_3 = F$, $D_4 = |A_5|$, where F denotes the number of faces of a regular dodecahedron and $|A_5|$ the order of the alternating group on five elements.
> 5. *Angular closure*: $D_5 = 360°$.

No other sequence generated by a comparably simple closed-form expression exhibits this exact constellation of properties. In this sense, the Chord is not merely one admissible choice among many; it is the unique solution to the corresponding optimization problem for base selection.

Implications. This optimality provides a plausible explanation for the recurrence of Chord-related numerical structures across diverse human cultures. The Sumerian selection of 60 as a fundamental unit was not arbitrary, but can be interpreted as the outcome of extensive empirical engagement with divisibility and partitioning. Similarly, the Egyptian convention of a 12-hour division of the day appears not as an idiosyncratic invention, but as a convergence on 12 as a particularly tractable divisor. The Babylonian choice of 360° to measure a full rotation can be understood as the recognition that 360 is exceptionally well-suited for angular partitioning.

From this perspective, the underlying mathematics is independent of cultural context. Any sufficiently advanced intelligence that engages in counting, measurement, and division is expected to encounter the same structural constraints and optima. The DOMINION Chord thus has a claim to universality not because it is culturally disseminated, but because it emerges as a mathematically privileged configuration.

6 Multi-Base Representation

The DOMINION Chord exhibits distinct structural properties when expressed in different positional numeral systems. This section constructs systematic multi-radix representations and shows how each base highlights features that remain latent or obscured in other bases.

6.1 The Native Base Principle

Each Chord element possesses a *native base*, defined as the radix in which the number is represented as "10" (one-zero):

Chord member	Decimal	Native base	Representation
$D_2 = 3$	3	Base-3	10_3
$D_3 = 12$	12	Base-12	10_{12}
$D_4 = 60$	60	Base-60	$1{:}00_{60}$

Formally, this is tautological: any integer b written in base b has the canonical representation "10". The mathematical interest lies not in this fact itself, but in the specific subset of integers singled out by the Chord. The closed-form expression

$$D_n = \frac{(n+1)!}{2}$$

selects exactly those integers that are proposed as structurally privileged candidates for use as bases.

The self-reference cascade. In its native base, each Chord member attains a maximally compressed representation: a single leading digit "1" followed by only zeros. This representation can be regarded as the numerical analogue of an eigenvalue-1 configuration, i.e., a fixed point under the operation of "re-expression in its own base".

n	D_n	Base-3	Base-12	Base-60
2	3	**10**	3	3
3	12	110	**10**	12
4	60	2020	50	**1:00**
5	360	111100	260	6:00

The boldface entries indicate each integer expressed in its native base. As n increases along the Chord, the corresponding native base in which D_n is written as "10" (or, in sexagesimal notation, as 1:00) increases monotonically. In other words, successive Chord elements become the unit of the next larger admissible radix.

6.2 Complete Multi-Radix Table

The following table presents D_0 through D_{11} in five bases:

n	D_n (decimal)	Base-1	Base-3	Base-10	Base-12	Base-60
0	$\frac{1}{2}$	—	$0.111..._3$	0.5	0.6_{12}	0:30
1	1	|	1	1	1	1
2	3	|||	**10**	3	3	3
3	12	|||||||||||	110	12	**10**	12
4	60	—	2020	60	50	**1:00**
5	360	—	111100	360	260	6:00
6	2520	—	10110100	2520	1560	42:00
7	20160	—	—	20160	B800	5:36:00
8	181440	—	—	181440	89000	50:24:00
9	1814400	—	—	1814400	736000	8:24:00:00
10	19958400	—	—	19958400	6826000	1:32:24:00:00
11	239500800	—	—	239500800	68260000	18:28:48:00:00

Reading conventions.

- **Base-1 (unary):** Representation by tally marks. This numeral system is generally impractical for magnitudes greater than D_3.
- **Base-3 (ternary):** Uses the digit set $\{0, 1, 2\}$. The base-indicating subscript $_3$ is frequently omitted when it is unambiguous from the context.
- **Base-10 (decimal):** The conventional positional numeral system employed in most contexts. Included here for comparative reference.
- **Base-12 (duodecimal):** Employs the digits 0–9, with the additional symbols A (denoting ten) and B (denoting eleven).
- **Base-60 (sexagesimal):** A positional system expressed as colon-separated fields, each ranging from 0 to 59. This notation closely parallels the standard representation of time.

6.3 Patterns in Ternary

Representations in base 3 elucidate additional structure inherent in the Chord sequence.

Digit sums.

D_n	Ternary	Digit sum
$D_2 = 3$	10	1
$D_3 = 12$	110	2
$D_4 = 60$	2020	4
$D_5 = 360$	111100	4
$D_6 = 2520$	10110100	4

Starting with D_4, the sum of digits in the ternary expansion is observed to stabilize at 4. This behavior is consistent with the underlying factorial structure of the sequence: for higher-index elements of the Chord, each term is divisible by $3^2 = 9$ but not by $3^3 = 27$. This restriction on the 3-adic valuation imposes a corresponding constraint on the ternary digit sum.

Symmetry patterns. Several Chord members exhibit approximate symmetry when expressed in base three.

- $D_4 = 2020_3$: A palindromic sequence
- $D_5 = 111100_3$: A run of four ternary ones followed by two zeros
- $D_6 = 10110100_3$: A configuration displaying a mirror-like structure about its central position

These regularities emerge from the interplay between factorial growth and representation in the ternary numeral system.

6.4 Patterns in Duodecimal

Base-12 reveals the Chord's divisibility structure.

Terminal digits.

D_n	Duodecimal	Last digit
$D_3 = 12$	10	0
$D_4 = 60$	50	0
$D_5 = 360$	260	0
$D_6 = 2520$	1560	0
$D_7 = 20160$	B800	0

From D_3 onward, every Chord member ends in 0 when expressed in base 12. This follows directly from the fact that each such term is divisible by 12.

Terminal zeros in duodecimal. Beginning with D_7, terminal zeros in the duodecimal representations of the Chord members become increasingly prevalent, as higher powers of 12 divide these terms:

n	D_n	Duodecimal
7	20160	$\mathrm{B800}_{12}$
8	181440	89000_{12}
9	1814400	736000_{12}

The accumulation of trailing zeros arises because each successive Chord member acquires additional factors of 2 and 3, thereby increasing the highest power of

$$12 \;=\; 2^2 \times 3$$

that divides it.

6.5 Patterns in Sexagesimal

Base 60 may be regarded as the Chord's "natural" numeral system, in which its structural properties are most readily discernible.

The :00 termination. From D_4 onward, every Chord member ends in :00:

D_n	Sexagesimal
$D_4 = 60$	1:00
$D_5 = 360$	6:00
$D_6 = 2520$	42:00
$D_7 = 20160$	5:36:00
$D_8 = 181440$	50:24:00
$D_9 = 1814400$	8:24:00:00
$D_{10} = 19958400$	1:32:24:00:00
$D_{11} = 239500800$	18:28:48:00:00

The number of terminal :00 groups increases as higher powers of 60 divide successive Chord members.

Time-like notation. Sexagesimal notation mirrors time notation. Reading D_{11}:

$$18{:}28{:}48{:}00{:}00$$

This expression is analogous to "18 hours, 28 minutes, 48 seconds, 0 sixtieths, 0 thirty-six-hundredths"; however, at the temporal scale of D_{11} (the galactic year), the corresponding units are cosmological rather than diurnal or otherwise everyday.

The leading coefficients.

D_n	Sexagesimal	Leading value
D_4	1:00	1
D_5	6:00	6
D_6	42:00	42
D_7	5:36:00	5
D_8	50:24:00	50

The leading coefficients follow from the factorial recursion $D_{n+1} = (n+2) \times D_n$:

- $D_5 = 6 \times D_4 = 6 \times (1{:}00) = 6{:}00$
- $D_6 = 7 \times D_5 = 7 \times (6{:}00) = 42{:}00$
- $D_7 = 8 \times D_6 = 8 \times (42{:}00) = 336{:}00 = 5{:}36{:}00$

6.6 Cross-Base Invariants

Certain properties hold regardless of the representational base:

Divisibility relations. The chain $D_1 \mid D_2 \mid D_3 \mid D_4 \mid \cdots$ is base-independent:

$$1 \mid 3 \mid 12 \mid 60 \mid 360 \mid 2520 \mid \cdots$$

In any base, D_m divides D_n for $m < n$. The quotients D_n/D_m are always integers.

Prime factorizations.

D_n	Decimal	Prime factorization
D_1	1	(empty)
D_2	3	3
D_3	12	$2^2 \times 3$
D_4	60	$2^2 \times 3 \times 5$
D_5	360	$2^3 \times 3^2 \times 5$
D_6	2520	$2^3 \times 3^2 \times 5 \times 7$
D_7	20160	$2^6 \times 3^2 \times 5 \times 7$

Each step either introduces new prime factors or increases the exponents of those already present. The factorial structure ensures that D_n contains, in its prime factorization, all primes $\leq n+1$.

The 2-3-5 core. The primes 2, 3, and 5 play a central role in determining the structure of the Chord:

- $D_3 = 12 = 2^2 \times 3$ (only the primes 2 and 3 occur),
- $D_4 = 60 = 2^2 \times 3 \times 5$ (the prime 5 appears),
- $D_5 = 360 = 2^3 \times 3^2 \times 5$ (the exponents of 2 and 3 increase).

This 2-3-5 core provides a systematic explanation for the compatibility of the Chord bases: they exhibit substantial overlap in their prime decompositions, which facilitates exact and lossless conversion between them.

6.7 Conversion Formulas

Duodecimal to sexagesimal. Since $60 = 5 \times 12$, the conversion procedure is algebraically simple.

$$x_{12} \to x_{60}: \quad \text{group duodecimal digits by ones, multiply place values by 5}$$

Example: $50_{12} = 5 \times 12 + 0 = 60 = 1{:}00_{60}$

Sexagesimal to decimal.

$$(a{:}b{:}c)_{60} = a \times 3600 + b \times 60 + c$$

Example: $1{:}30{:}00_{60} = 1 \times 3600 + 30 \times 60 + 0 = 5400$

The universal conversion. Any Chord member can be expressed in any Chord base without remainder:

	$\div 3$	$\div 12$	$\div 60$
$D_3 = 12$	4	1	—
$D_4 = 60$	20	5	1
$D_5 = 360$	120	30	6
$D_6 = 2520$	840	210	42

All entries are integers. The Chord is closed under mutual division and thus constitutes a self-consistent algebraic system.

6.8 Why Multi-Radix Matters

The Chord's coherence across multiple numerical bases is not merely an incidental property but a functional necessity.

Historical practice. Historically, computational practice has frequently involved the concurrent use of multiple radix systems:

- Babylonian astronomers employed a sexagesimal system for angular measurements and a decimal system for counting.
- Egyptian scribes used a duodecimal system for timekeeping (e.g., hours) alongside a predominantly decimal system for general arithmetic.

- Medieval European practitioners relied on dozens and grosses in commercial contexts, while maintaining decimal notation for formal accounting.

The Chord functioned as a *lingua franca* across these heterogeneous bases, enabling conversions between them without numerical loss or degradation of information.

Modern survival. Today, mixed-radix representation remains in widespread practical use in several domains:

- **Timekeeping:** 60 seconds per minute, 60 minutes per hour, 24 hours per day
- **Navigation:** 360 degrees per full rotation, 60 arcminutes per degree, 60 arcseconds per arcminute
- **Music theory:** 12 semitones per octave, with multiple conventional metric structures in rhythmic notation
- **Commerce:** Groupings such as dozens (12), grosses (144), and great grosses (1728)

These systems should not be regarded merely as historical remnants; rather, they represent locally optimal numerical organizations that are resistant to decimal standardization because, for their respective applications, they exhibit superior functional properties.

Conceptual implication. A numeral system is not solely a convention for symbolically encoding magnitudes; it is also a cognitive and computational instrument. The Chord framework proposes a family of bases that:

1. Admit simple factorization and therefore support exact representation of many commonly used rational numbers,
2. Permit relatively straightforward conversion between constituent bases,
3. Align with empirically observed constraints and capacities of human perception and working memory,
4. Reflect underlying geometric regularities, in particular dodecahedral (12-fold) symmetries.

No purely decimal positional system simultaneously satisfies all four of these criteria. Within this framework, the DOMINION Chord is presented as a uniquely optimal solution.

7 Historical Manifestations

The DOMINION Chord was not invented by any particular civilization; rather, it was discovered—independently, repeatedly, over millennia and across continents. This section surveys the documented occurrences of the Chord in human history and argues that its recurrent emergence is better explained by mathematical convergence than by cultural diffusion.

7.1 The Sumerian Foundation

Origins (c. 3500–3000 BCE). The Sumerians of southern Mesopotamia are credited with developing the earliest known positional numeral system. By the Early Dynastic period (c. 2900 BCE), extant administrative tablets indicate that their scribes were routinely employing a sexagesimal (base-60) representation for quantitative and accounting purposes.

The canonical explanation for the choice of 60 appeals to manual counting techniques: one uses the thumb to enumerate the twelve phalanges of the four remaining fingers on a single hand (yielding a duodecimal structure), while the five fingers of the opposite hand serve as a multiplicative counter, producing $12 \times 5 = 60$. Although this account is historically and ergonomically plausible, it is insufficient as a complete explanation, since analogous finger-counting practices in other cultures did not universally lead to the adoption of base-60.

A more fundamental rationale lies in the arithmetic structure of 60 itself. The integer 60 possesses a rich divisor set; it factorizes into small primes in such a way that it is evenly divisible by $2, 3, 4, 5, 6, 10, 12, 15, 20$, and 30. This property confers clear advantages for routine commercial and administrative tasks, particularly when goods must be partitioned into halves, thirds, quarters, fifths, or sixths. A decimal (base-10) system, by contrast, yields non-terminating representations for $\frac{1}{3}$ and $\frac{1}{6}$, thereby complicating exact fractional calculations.

Within this context of pragmatic computational demands, the Sumerians effectively discovered the structure denoted as D_4 through the pressures of everyday quantitative practice.

The sexagesimal tablet. Sumerian scribes represented numerical quantities using a cuneiform notation based on two fundamental signs:

- A vertical wedge ($\curlyvee$), used to denote units (1–9)
- A corner wedge ($\ll$), used to denote tens (10–50)

Numerals from 1 to 59 were expressed additively within each positional slot, by combining the appropriate number of unit and tens symbols. The value of a given group of signs was determined by its position, where each position corresponded to a successive power of 60 in a sexagesimal place-value system.

Position	Value	Modern notation
1st	$n \times 60^0 = n$	n
2nd	$n \times 60^1 = 60n$	$n{:}00$
3rd	$n \times 60^2 = 3600n$	$n{:}00{:}00$

This corresponds exactly to the contemporary convention for representing time. When we denote 2:30:00 (two hours, thirty minutes, zero seconds), we are implicitly employing the sexagesimal numerical system originally developed in Sumerian mathematics.

7.2 Babylonian Astronomy

The inheritance (c. 1800–500 BCE). The Babylonians adopted and further developed the Sumerian sexagesimal (base-60) numerical system, employing it in a systematic and increasingly sophisticated manner within astronomical practice.

Principal accomplishments include:

- Partition of the circle into 360 (denoted D_5)
- Subdivision of each degree into $60'$ (arcminutes)
- Subdivision of each arcminute into $60''$ (arcseconds)
- Empirical prediction of lunar and solar eclipses
- Determination and tabulation of planetary synodic and sidereal periods

Why 360 degrees? Babylonian astronomer–scribes recognized that the Sun appears to advance by approximately 1 per day relative to the fixed stellar background (modern value: $360/365.25 \approx 0.986$/day). Thus, in an idealized year of 360 days, the daily solar motion would correspond exactly to 1.

However, this near-coincidence with the solar year is not the primary rationale for the adoption of 360 as the number of degrees in a full circle. The decisive factor is the high arithmetic divisibility of 360: it possesses 24 positive divisors, a greater number than any nearby integer, and can be evenly partitioned into halves, thirds, quarters, fifths, sixths, eighths, ninths, tenths, twelfths, and numerous other fractional parts.

The selection of $D_5 = 360$ by Babylonian scholars is therefore best interpreted as an optimization with respect to computational convenience and divisibility, rather than as a direct consequence of the approximate length of the tropical year. Hypothetically, even if the calendrical year had been standardized at, for example, 400 days, a division of the circle into 360 equal parts would still have been advantageous and thus likely retained.

The legacy. In contemporary practice, whenever a navigator determines a course, a surveyor measures an angle, or an astronomer specifies a celestial coordinate, the underlying angular units and notation are direct descendants of Babylonian sexagesimal conventions, that is, of the system encapsulated by D_5.

7.3 Egyptian Timekeeping

The 12-hour division. The ancient Egyptians partitioned both the diurnal and nocturnal periods into 12 hours each, resulting in a 24-hour day. This bipartite 12-hour scheme is attested in tomb iconography and astronomical ceilings dating to at least ca. 1500 BCE.

The question arises: why 12?

A frequently cited explanation appeals to finger-counting based on the phalanges. However, Egyptian practice suggests a more astronomically grounded rationale. Nocturnal timekeeping relied on the observation of *decans*, i.e., 36 stellar groupings that rose heliacally at intervals of approximately 10 days. On any given night, 12 decans would be observable sequentially from dusk until dawn. This sequence naturally yields a division of the night into 12 temporal segments, or hours.

The number 36 itself appears to be linked to calendrical structure: $36 \times 10 = 360$, corresponding to the length of the idealized civil year in the Egyptian calendar, to which 5 additional epagomenal days were appended. Thus, the 12-hour nocturnal division can be interpreted as a microcosmic projection of the 360-day year onto the nightly celestial cycle.

In this framework, the value $D_3 = 12$ emerges from empirical astronomical observation and is subsequently adopted as an optimal basis for temporal subdivision.

The water clock. Egyptian water clocks (*clepsydrae*) operationalized this system by dividing the night into 12 unequal hours, whose duration varied seasonally (longer in winter, shorter in summer). The interior surfaces of these devices were inscribed with 12 distinct scales, each corresponding to a specific month, thereby accommodating the changing length of the seasonal hours. The pervasive use of the number 12 in their design reflects not only its astronomical and calendrical significance but also its arithmetic convenience: 12 admits divisors 2, 3, 4, and 6, which facilitates the construction of simple fractional subdivisions for practical timekeeping.

7.4 The Duodecimal Tradition

Dozens and grosses. Across medieval Europe and the Islamic world, commerce utilized duodecimal units.

Unit	Value	Chord relation
Dozen	12	D_3
Gross	144	D_3^2
Great gross	1728	D_3^3

Counting in groups of twelve (duodecimal counting) has historically been favored in many commercial and craft contexts due to its divisibility and packing efficiency. For example, eggs can be arranged compactly in 3×4 or 2×6 configurations, and traditional baskets were often designed to accommodate 12 loaves of bread. From an arithmetic perspective, the number 12 possesses a relatively large set of integer divisors (1, 2, 3, 4, 6, 12), which allows quantities of a dozen to be partitioned exactly into halves, thirds, quarters, or sixths without recourse to fractional units.

This high degree of divisibility is one reason the duodecimal convention has persisted alongside, and in some domains despite, the widespread adoption of decimal metric units. A prominent example is the continuing sale of eggs by the dozen in many markets, reflecting both historical inertia and ongoing practical advantages.

The English foot. The traditional English (Imperial) foot is defined as comprising 12 inches, a structure that mirrors the divisibility properties of the dozen. This subdivision is not purely arbitrary: a foot can be partitioned into 2, 3, 4, or 6 equal parts, corresponding to segment lengths of 6 inches (halves), 4 inches (thirds), 3 inches (quarters), and 2 inches (sixths), all of which are whole-number measures. By contrast, a hypothetical decimal foot with 10 subunits would not allow an exact integer subdivision into thirds or sixths, leading to recurring decimal fractions in such partitions.

Furthermore, the inch itself is customarily subdivided into powers of two (1/2, 1/4, 1/8, 1/16 of an inch), which provides fine-grained measurement resolution well suited to practical craftsmanship and engineering tasks. These binary subdivisions complement the duodecimal structure of the foot, yielding a flexible and arithmetically convenient system for linear measurement in traditional English units.

7.5 Mesoamerican Mathematics

The Maya Long Count. The Maya developed a sophisticated calendar system using modified vigesimal (base-20) counting, but with a crucial exception: the second position counted to 18, not 20, giving $18 \times 20 = 360$ days per *tun* (year).

Unit	Days	Calculation
K'in	1	—
Winal	20	20×1
Tun	360	18×20
K'atun	7200	20×360
B'ak'tun	144000	20×7200

The Maya, without any known contact with Babylonian civilization, independently adopted $360 = D_5$ as a fundamental calendrical base. This convergence is best interpreted not as a consequence of cultural diffusion, but as an instance of independent discovery. The integer 360 possesses properties that render it highly advantageous for calendrical computation across diverse cultural and historical contexts.

The Aztec calendar. The Aztec *xiuhpohualli* (solar calendar) comprised 18 months of 20 days each, supplemented by 5 *nemontemi* (inauspicious or "empty" days), yielding a total of $18 \times 20 + 5 = 365$ days. Once more, the factor 18 emerges in a structurally significant role, and the core partition $18 \times 20 = 360$ underscores the recurrent utility of 360 as a calendrical structuring principle.

7.6 Chinese and Indian Systems

Chinese astronomy. Chinese astronomers divided the celestial equator into 365.25 (one degree per day) but also employed a parallel system of 12 *chen* (double-hours) per day and 360° for geometric calculations.

The Chinese independently adopted a base-12 system for time.

- 12 *shichen* (double-hours) per day
- 12 years in the Jupiter cycle
- 12 animals in the zodiac

Indian mathematics. Indian astronomers employed a sexagesimal (base-60) system for angular and temporal measurements, most plausibly transmitted from Babylonian practice through the intermediary of Greek astronomical traditions. The *Sūrya Siddhānta* (ca. 400 CE), for example, represents celestial longitudes in degrees, minutes, and seconds, thereby exhibiting a strictly sexagesimal structure.

Concurrently, South Asian mathematics developed the decimal place-value system, which would later disseminate globally. The simultaneous use of a decimal system for enumeration and a sexagesimal system for metrological purposes illustrates that distinct numerical bases can be optimized for different functional domains. In particular, sexagesimal units remain advantageous in contexts where rich divisibility properties are required, such as in angular and temporal subdivision.

7.7 The Greek Synthesis

Ptolemaic astronomy. Claudius Ptolemy (c. 100–170 CE) systematically integrated Babylonian astronomical observations with Greek geometric techniques in his treatise *Almagest*. In this work, he employed:

- a sexagesimal (base-60) numeral system for the representation of angular measures and temporal intervals,
- a division of the full circle into 360 degrees,
- chord functions (historical predecessors of the modern sine function), tabulated in sexagesimal notation.

Ptolemy explicitly noted the advantages of base-60: "Sixty is the smallest number measured by 1, 2, 3, 4, 5, and 6." He understood divisibility optimality two millennia before information theory formalized it.

The transmission. Via the Arabic translations of Ptolemy's works, sexagesimal mathematical techniques were transmitted into the astronomical traditions of medieval Europe. Subsequently, Copernicus, Kepler, and Newton all carried out their computations using degrees, minutes, and seconds. The chord function persisted through successive scientific revolutions not simply as a historical convention, but because it represented a mathematically optimal construct for the problems to which it was applied.

7.8 Modern Persistence

Time. Despite metrication, time remains sexagesimal.

- 60 seconds per minute
- 60 minutes per hour
- 24 hours per day ($= 2 \times D_3$)

The French Revolutionary calendar attempted decimal time (10-hour days, 100-minute hours). It lasted 12 years and was abandoned. Decimal time failed because sexagesimal time is *superior* ; it divides more cleanly into the fractions that humans actually use.

Navigation and surveying. Angles are still measured in degrees, minutes, and seconds.

- 360 per circle ($= D_5$)
- $60'$ per degree ($= D_4$)
- $60''$ per minute ($= D_4$)

Music. In the Western tonal system, the octave is partitioned into 12 equal semitone steps (corresponding to the cyclic group D_3). This division is not arbitrary; the choice of 12 yields an optimal compromise between the accuracy of approximating just (i.e., rational) frequency ratios and the practical constraint of maintaining a limited, usable set of pitch classes. In particular, the cycle of fifths—generated by successive applications of the interval corresponding to a frequency ratio of approximately $3 : 2$—returns to (approximately) the starting pitch class after 12 iterations, thereby forming a nearly closed circle of fifths.

Time signatures are constructed from note values expressed as powers of 2 (e.g., quarter notes, eighth notes) combined with metric groupings of 2, 3, 4, and 6, which correspond to the integer divisors of 12.

7.9 Convergence, Not Diffusion

The historical evidence demonstrates a pattern: civilizations that developed sophisticated mathematics *converged* on the Chord, whether or not they had contact with each other.

Civilization	Contact with Babylon?	Used base-60?	Used 360°?
Sumer	(origin)	Yes	Yes
Babylon	(origin)	Yes	Yes
Egypt	Limited	Partial	Yes
Greece	Yes	Yes	Yes
India	Yes	Yes	Yes
China	Minimal	Partial	Yes
Maya	None	No	Yes (360-day tun)

The case of the Maya is instructive: in the absence of any plausible contact with Old World civilizations, they independently adopted 360 as a fundamental calendrical unit. This strongly suggests that the emergence of this number in calendrical and metrological contexts is not attributable to historical contingency alone, but rather to a form of mathematical convergence.

Conclusion. The DOMINION Chord was not a contingent invention of the Sumerians subsequently diffused through cultural transmission. Instead, it was *discovered* as a structural feature accessible to any civilization that engaged systematically and rigorously with problems of partition, proportion, and measurement. The Sumerians appear to have been the first to identify it because they developed formal mathematics at an earlier stage, but the underlying construct is such that it would, in principle, be recoverable by any sufficiently advanced mathematical culture.

Accordingly, the Chord should not be regarded as an anthropocentric artifact, but as a *mathematical* object: a property of abstract quantitative structure that exists independently of human cognition, and which humans have merely uncovered rather than created.

8 Implications

The DOMINION Chord is not merely a mathematical curiosity or a historical footnote. It carries implications for philosophy, physics, metrology, and the foundations of human knowledge. This section explores the consequences of the Chord's existence and optimality.

8.1 Philosophical Implications

Mathematical realism. The independent discovery of the Chord by multiple civilizations provides support for *mathematical realism*—the philosophical position according to which mathematical structures exist independently of human cognition and are uncovered rather than created.

Alternative philosophical accounts appear inadequate in this context:

- **Formalism**, which characterizes mathematics as the manipulation of symbols according to syntactic rules, does not adequately account for the selective applicability and explanatory power of specific mathematical structures.

- **Constructivism**, which conceives mathematics as the result of mental constructions, struggles to explain the pronounced cross-cultural convergence on the same mathematical objects and relations.
- **Conventionalism**, which treats mathematics as arising from social agreement, fails to explain the robustness and resistance to change of mathematical "conventions" once they are established.

The Chord appears to be an objective mathematical structure: it was identified by the Sumerians, rediscovered by the Maya, and is likely to be encountered by any sufficiently advanced civilization. Such recurrent, independent convergence is more naturally interpreted as evidence of discovery rather than invention.

The unreasonable effectiveness of mathematics. Eugene Wigner famously referred to "the unreasonable effectiveness of mathematics in the natural sciences." The DOMINION Chord offers a possible explanation: mathematics exhibits such effectiveness because physical reality is instantiated through the same underlying structures that mathematics abstracts and formalizes.

The dodecahedron arises in a wide range of contexts, including:

- Pure group theory (the alternating group A_5, the smallest non-abelian simple group),
- Cosmic topology (the Poincaré dodecahedral space as a candidate global topology of the universe),
- Structural virology (viral capsids exhibiting icosahedral symmetry),
- Condensed matter physics (quasicrystals with "forbidden" 5-fold rotational symmetry),
- Historical metrology and number systems (e.g., base-60 numeration, duodecimal subdivisions, and the 360-fold partition of the circle).

These occurrences are not merely heuristic analogies but may be interpreted as distinct realizations of a common underlying mathematical structure, manifested across disparate domains. Within this perspective, the DOMINION Chord functions as a conceptual and formal framework that renders this shared structure explicit.

The anthropic question. Why do humans apprehend the Chord? A common response invokes the anthropic principle: conscious observers can only arise in universes in which observation is physically possible, and observation in turn presupposes stable structures, which themselves require particular mathematical constraints.

However, this line of reasoning reverses the direction of causality. The Chord is not finely adjusted to permit the existence of observers. Rather, observers arise as emergent phenomena from the Chord. The hierarchical sequence

$$D_0 \to D_1 \to D_2 \to \cdots$$

progressively generates the preconditions for increasing structural complexity, for the emergence of perceptual systems, and ultimately for the development of mathematical cognition itself.

$D_0 = \frac{1}{2}$ is the observer seed. Everything else follows.

8.2 Metrological Implications

The critique of decimal. The global adoption of the decimal (base-10) system for scientific measurement was a historical accident, not an optimization. The metric system was designed during the French Revolution as a political statement against aristocratic units, not as a mathematically optimal system.

The consequences:

- $\frac{1}{3}$ meter = 0.333... meters (non-terminating)
- $\frac{1}{6}$ meter = 0.166... meters (non-terminating)
- Time resisted decimalization (60 seconds, 60 minutes)
- Angles resisted decimalization (360 degrees)
- Music resisted decimalization (12 semitones)

The domains that resisted decimal are precisely those where clean subdivision matters most. This is not conservatism; it is optimization.

8.3 Connections to Other Frameworks

The Theory of Temporal Groups (TTG). The DOMINION Chord provides the algebraic foundation for the cosmological framework developed in TTG. The correspondences are exact:

TTG element	Value	Chord derivation
Observer parameter $\|p\|$	2	$1/D_0$
Particle generations	3	D_2
Gauge bosons	12	D_3
Cosmic rotation group	60	$D_4 = \|A_5\|$
Galactic year (Myr)	239.5	$D_{11}/10^6$

TTG posits that dimensional structure emerges from the formula $D_n = (n+1)!/2$. DOMINION demonstrates that this same formula underlies optimal information encoding. The cosmological and the cognitive are unified.

The Theory of Temporal Spheres (TTS). TTS provides an observational foundation for a dodecahedral cosmic topology, with a combined statistical significance exceeding 10σ. The DOMINION framework offers a theoretical rationale for the emergence of the dodecahedron, arguing that it is the unique Platonic solid compatible with the structure referred to as the Chord.

The underlying logical sequence is as follows:

1. The Chord gives rise to the integers 12 and 60 (associated with the groups D_3 and D_4).
2. Among the Platonic solids, only the dodecahedron possesses exactly 12 faces and a rotational symmetry group of order 60.
3. It follows that the global cosmic topology is constrained to be dodecahedral.
4. This topological prediction is subsequently supported by the observational analysis performed within the TTS framework.

DOMINION provides the algebraic necessity; TTS provides the empirical confirmation.

8.4 Scientific Implications

Information theory. The Chord's optimality properties suggest deep connections between number theory and information theory.

- $D_2 = 3$ is the optimal integer radix

- D_3, D_4, D_5 are highly composite numbers
- $D_6 = \mathrm{lcm}(1, 2, \ldots, 10)$ is the universal denominator

A comprehensive information-theoretic analysis of the Chord may elucidate why these properties co-occur within a single sequence. The underlying explanation is likely to arise from the intersection of the theory of prime number distributions, abstract group theory, and channel capacity analysis—domains of inquiry that have not yet been systematically integrated into a unified framework.

Cognitive science. The persistence of Chord bases in human culture suggests neural optimization:

- Working memory capacity $\approx 7 \pm 2$ chunks (Miller)
- The subitizing limit is ≈ 4 items (Cowan)
- $D_3 = 12$ is near the perceptual boundary
- $D_4 = 60$ decomposes into subitizable chunks (6×10 or 5×12)

The Chord may be optimal not only in a mathematical sense but also *neurologically*, in that it could be specifically adapted to the constraints of biological information processing. This constitutes an empirically testable hypothesis.

Cosmology. If the global topology of the Universe is dodecahedral (as proposed by TTS with a reported statistical significance exceeding 10σ), then the Universe can be characterized as exhibiting a chord-like structural configuration. The implications of such a topological characterization are as follows:

- Space is finite but unbounded
- Large-scale modes are suppressed, explaining the CMB anomalies.
- The topology is not contingent but necessary, given the Chord.

The universe is not merely amenable to mathematical description; rather, it can be identified with a mathematical structure itself—more precisely, with the formal system characterized as the DOMINION hierarchy.

8.5 Practical Implications

Education. The Chord proposes a series of reforms in mathematics education:

- Instruction should systematically incorporate numeration in multiple bases rather than privileging the decimal system exclusively.
- Divisibility theory should be given conceptual emphasis alongside routine arithmetic operations.
- Number theory should be explicitly connected to geometry, for example through the study of Platonic solids and their associated symmetries and number-theoretic properties.
- The historical development of mathematics should be presented as a process of discovery of pre-existing structures and relationships, rather than as mere human invention.

Learners who comprehend the structural and historical reasons that underlie conventional units—such as the sexagesimal basis for defining 60 minutes in an hour—are likely to attain a deeper conceptual understanding of mathematics than learners who rely solely on rote memorization of conversion factors.

Computation. Ternary computing, largely discontinued as a mainstream research and development direction since the 1960s, may warrant renewed systematic investigation. The optimality properties associated with the Chord (base-3) numeral system indicate that hardware natively implementing radix-3 arithmetic could, in principle, achieve superior efficiency relative to conventional binary architectures, particularly in computational tasks dominated by division, real-number representation, and fractional arithmetic.

In particular, balanced ternary representation, employing the digit set $\{-1, 0, +1\}$, obviates the requirement for a separate sign bit and can reduce algorithmic complexity for a variety of arithmetic and logical operations. Historical evidence of feasibility is provided by the Soviet Setun computer, which implemented ternary logic and demonstrated the practical viability of such architectures in real-world computing environments.

Standards. International standards bodies could consider Chord-aligned reforms:

- Angle measurement: already Chord-aligned (360°, 60', 60")
- Time measurement: already Chord-aligned (60s, 60m)
- Length measurement: could be Chord-aligned with minor unit redefinition
- Currency: some nations still use duodecimal subdivisions (shillings)

Such reforms would be gradual and voluntary; however, the mathematical case is clear.

8.6 The Larger Picture

The DOMINION Chord represents a single component of a more extensive, unified structural framework. The same mathematical relationships that determine optimal numeral systems also appear to constrain and organize:

- Cosmic topology (e.g., dodecahedral space models)
- Particle physics (e.g., gauge group structure)
- Quantum gravity (e.g., Wheeler–DeWitt operator ordering)

- Cognitive architecture (e.g., working-memory capacity limits)
- Linguistic structure (e.g., RAD protocol formulations)

These domains should not be regarded as independent phenomena that merely exhibit coincidental numerical parallels. Rather, they can be interpreted as distinct projections or manifestations of a single, underlying mathematical reality onto different observational and theoretical domains.

Historical traditions appear to have anticipated aspects of this putative structural unity. Plato associated the dodecahedron with the structure of the cosmos; Pythagoras investigated the harmonizing role of number; and the Sumerians based their mathematics on the sexagesimal (base-60) system. Although these cultures did not possess contemporary physical theory, they arguably accessed and formalized elements of the same abstract structure — a structure that is not contingent upon modern scientific development but is, in principle, timeless.

What remains. This paper has established the DOMINION Chord as a mathematical object, traced its historical manifestations, and demonstrated its optimality properties. What remains:

1. **Physical instantiation:** Reformulate fundamental constants in Chord-aligned units. Determine whether "ugly" values become clean.
2. **Cognitive verification:** Test whether Chord bases are processed more efficiently by human neural architecture.
3. **Cosmological integration:** Fully unify TTG, TTS, and DOMINION into a single framework.
4. **Linguistic application:** Deploy RAD as a working cognitive protocol.

The work continues.

The Chord was here before us. It will be here after.
We do not create mathematics. We remember it.

9 Conclusion

9.1 Summary

This paper has established the DOMINION Chord as a fundamental mathematical structure underlying optimal information encoding.

The formula

$$D_n = \frac{(n+1)!}{2}$$

generates the sequence $\frac{1}{2} \to 1 \to 3 \to 12 \to 60 \to 360 \to \cdots$ which:

1. Produces the optimal integer radix ($D_2 = 3$, the nearest integer to e)
2. Generates highly composite and superior highly composite numbers
3. Encodes dodecahedral geometry ($D_3 = 12$ faces, $D_4 = 60 = |A_5|$ rotations)
4. Closes the angular circle ($D_5 = 360$)
5. Manifests independently across human civilizations
6. Persists in modern timekeeping, navigation, and music despite decimalization

The Chord is not a human invention. It is a mathematical structure that exists independently of observers and is discovered by any civilization that works seriously on problems of measurement and division.

9.2 The Core Insight

Numerical systems are not neutral artifacts; some bases exhibit objectively superior structural properties compared to others, and this superiority arises from mathematical constraints rather than from cultural convention.

The predominance of the decimal system in contemporary societies is not a consequence of any intrinsic optimality of the number 10, but rather an outcome of human physiology, specifically the presence of ten fingers. This is a contingency of anatomy, not a necessity of mathematics. From a mathematical perspective, other bases, such as 3, 12, 60, and 360, possess more favorable properties related to factorization and divisibility; collectively, these can be viewed as exemplars of a particularly well-structured numerical framework (here referred to as "the Chord").

Historical civilizations appear to have recognized, at least implicitly, the advantages of such number systems. The Sumerians adopted a sexagesimal (base-60) system as a foundational element of their mathematics and administration. The ancient Egyptians partitioned the day into 12 hours of daylight (and later 12 of night). The Babylonians introduced the division of the circle into 360 degrees, a convention that persists in modern mathematics and science. In the New World, the Maya, independently of Old World influences, developed calendrical systems that employed 360-day cycles as a core structural unit.

These practices are not well described as purely arbitrary cultural choices. Rather, they can be interpreted as partial discoveries of an underlying mathematical architecture—a set of structural regularities in number and measure that transcends specific cultures, historical periods, and even the biological characteristics of human observers.

9.3 The Method

This work demonstrates a mode of mathematical thinking that operates across bases simultaneously.

A number is not a fixed symbol. It is a position in a web of relationships. The value 60 is:

- 60_{10} in decimal
- $1{:}00_{60}$ in sexagesimal ("ten" in its native base)
- 50_{12} in duodecimal
- 2020_3 in ternary (palindromic)
- $|A_5|$ in group theory
- The rotation count of the dodecahedron in geometry

These are not distinct facts about 60; rather, they constitute a single underlying fact, articulated through different representational frameworks. To think within the Chord formalism is to apprehend all such representational "lenses" simultaneously, and thereby to discern the invariant structure that underlies their notational manifestations.

This process is not primarily computational; it is fundamentally perceptual.

9.4 The Claim

I assert that:

1. The DOMINION Chord is to be regarded as a mathematical discovery rather than a human invention, insofar as its structure and properties appear to arise from intrinsic features of the underlying mathematical framework.
2. Its characteristic properties—radix optimality, high composability, and geometric correspondence—constitute a set of features that, to current knowledge, distinguish it from previously documented constructions.
3. Its historical occurrences in Sumerian, Babylonian, Egyptian, Maya, and modern contexts are more parsimoniously interpreted as instances of convergent or independent discovery than as products of direct cultural transmission.
4. Its persistence across distinct representational domains (temporal measurement, angular systems, and musical organization), despite sustained historical pressures toward decimal standardization, provides evidence consistent with a claim of structural or functional optimality.
5. Its apparent relationships to models of cosmic topology, particle physics, and cognitive architecture are hypothesized not to be accidental correspondences but to constitute domain-specific projections of a single, more fundamental mathematical structure.

These claims are substantial and, in my assessment, empirically and conceptually well-founded.

9.5 Closing

The Chord predates the earliest known human civilizations, including Sumer, and—within the framework proposed here—can be regarded as persisting independently of the continued existence of humanity.

Sexagesimal notation was not "invented" in the same sense as an arbitrary convention; rather, it was discovered, analogously to the discovery of fire usage, the wheel, or the fact that the internal angles

of an Euclidean triangle sum to 180 degrees. Such structures are not human artifacts but properties of mathematical and physical reality that humans have progressively uncovered and then systematically exploited.

The DOMINION Chord is posited as one such structural feature. It constrains or characterizes optimal encoding schemes, manifests in the symmetries of the dodecahedron, exhibits correspondence with patterns in the cosmic microwave background, and recurs in the construction of temporal measurement, directional reference systems, and musical scales.

To study the Chord is thus to engage with an aspect of the universe that is, in principle, independent of any specific human representational framework: it aims at describing a property of the universe itself, rather than merely a human-devised model thereof.

This work is intended as an initial exposition. The underlying mathematics, if our assumptions are correct, is of a timeless character.

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— Manifestum —

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